Numerical Evaluation of Neoclassical Transport in Stellarators at Arbitrary Collisionality

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Z. Naturforsch. **37a**, 906–911 (1982); received May 28, 1982

To Professor Arnulf Schlüter on his 60th Birthday

The neoclassical diffusion losses of an $l=2$ stellarator and an advanced stellarator are analysed with a modified version of the Boozer-Kuo-Petravic Monte-Carlo Code. Particle loss and confinement of a monoenergetic ion distribution are determined at arbitrary collisionality under stationary conditions. Special attention is given to the long mean free path regime. Due to the drift motion of localized particles a depopulation of the distribution function around $v_\parallel = 0$ occurs. The confinement time is determined by pitch angle scattering in this case.

It is analysed under which conditions the $1/\nu$-scaling of diffusion losses arises. A comparison is made between the loss rate of an advanced stellarator and a classical $l=2$-stellarator. Also a tokamak with ripple losses is considered.

I. Introduction

Neoclassical diffusion in stellarators has been investigated by many authors both analytically and numerically. Due to the localized particles trapped in helical mirrors a strong increase of diffusion losses may occur in the regime of low collisionality. For stellarators this theory has been worked out by Connor and Hastie [1], Frieman [2] has shown that the $1/\nu$-scaling of diffusion losses holds for general 3 dimensional configurations with localized particles. Recently the diffusion losses in a stellarator have been computed by Monte-Carlo-techniques by Boozer and Kuo-Petravic [3] and Mynick [4]. In these papers the analytical theory was confirmed; on the other hand, however, numerical studies done by Potok et al. [5] for a torsatron configuration found much lower neoclassical losses than predicted by theory. This discrepancy is still an unsolved problem.

As has already been shown by Miyamoto [6] the losses decrease again with collision frequency at very low collisionality. In this regime the losses are determined by velocity space diffusion into the loss cone and by the unperturbed drift of these particles. This fact demonstrated that the numerical techniques, which are used in the above mentioned papers in order to calculate a spatial diffusion coefficient are not applicable to the very long mean free path regime. The collisional losses in this regime are determined by random walk in velocity space rather than in real space.

Also some criticism has to be made about the analytic theories. The two scaling parameters of neoclassical theory are λ/L_0 and a/ϱ with $\lambda=$ mean free path, $\varrho=$ gyroradius, $a=$ plasma radius, L_0 is a length, which in the tokamak is chosen to be R/ι (connection length).* For convenience it will also be used in the present considerations.

This means that the solution of the drift kinetic equations is a general function of $\boldsymbol{x}, E, \mu, \lambda\iota/R, a/\varrho$, ($E=$ energy, $\mu=$ magnetic moment). The solution not only depends on the collisionality $(\lambda\iota/R)^{-1}$ but also on the number of gyroradii in the plasma radius. According to the special choice of these parameters different approaches to solve the drift kinetic equation have to be made. The $1/\nu$ scaling of losses occurs if the bounce time of a localized particle is longer than the detrapping time ($\tau_{\mathrm{Bounce}} > \varepsilon_{\mathrm{h}}/\nu$, $\varepsilon_{\mathrm{h}} = \delta B/B$ of the helical ripple, $\nu=$ collision frequency). This happens if $\lambda\iota/R > (L\iota/R)(1/\varepsilon_{\mathrm{h}}^{3/2})$, L is the length of the helical ripple. Although collisions are rare in this regime theory assumes a local Maxwellian as a lowest order approximation to the distribution function. On the other hand if the drift of localized particles towards the wall is fast enough, a Maxwellian no longer can be maintained. This situation occurs if the drift time $\tau_{\mathrm{Drift}} = a/v_{\mathrm{Drift}} \approx a\,R/\varrho\, v_{\mathrm{th}}$ is smaller than the $90°$ collision time ν^{-1} ($v_{\mathrm{th}}=$ thermal velocity). Therefore the loss of localized particles causes a deviation

* Definition: $2\pi\iota$ = angle of rotational transform.

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from an isotropic Maxwellian and consequently diffusion in velocity space if $\lambda\iota/R > (a/\varrho)\iota$. If a/ϱ is not too large this limit can be reached before the onset of the $1/\nu$-scaling. It has to be expected that under these conditions a rather smooth transition from the plateau regime to the regime of the ν-scaling occurs.

In this paper the transition region is investigated with the Monte-Carlo-technique developed by Boozer et al. [3]. But instead of the diffusion coefficient the particle flux to the wall and the confinement time is calculated in stationary conditions. By calculating the loss flux instead of the diffusion coefficient, the drift motion of trapped particles is included properly. This method includes both diffusion in velocity space and in real space.

II. The Model

The procedure follows closely the methods developed by Boozer et al. The orbits of the charged particles are described in the natural coordinate system (ψ, χ, φ_0) with

$$\boldsymbol{B} = \nabla\varphi_0 \times \nabla\psi = \nabla\chi\,. \tag{1}$$

ψ is the flux function describing the magnetic surfaces, χ the scalar potential of the magnetic field and $\varphi_0 = \text{const}$ along the lines of force. In order to correlate these coordinates to the conventional coordinate system (r, θ, φ) (r = distance from the magnetic axis, θ = poloidal angle, φ = toroidal angle) several approximations are used

$$\psi = \tfrac{1}{2} r^2\,, \quad \varphi_0 = \iota\varphi - \theta\,, \quad (a/R)\chi = \varphi\,. \tag{2}$$

In this approximation the flux tubes are axisymmetric tori with circular cross section. The pitch $\mathrm{d}\theta/\mathrm{d}\varphi = \iota$ of a field line is constant. The different configurations differ in

$$B = B(r, \theta, \varphi) \quad \text{or} \quad B = B(\psi, \varphi_0, \chi)$$

Tokamak

$$B = B_0\left(1 - \frac{r}{R}\cos\theta\right). \tag{3}$$

Stellarator

$$B = B_0\left(1 - \frac{r}{R}\cos\theta - \delta_a\left(\frac{r}{a}\right)^l \cos(l\theta - m\varphi)\right), \tag{4}$$

l and m are integers, $\delta_a = \text{const}$.

Advanced stellarator

$$B = B_0\left(1 - \frac{r}{R}\cos\theta\,(1 - \varepsilon_b \cos m\varphi) - \varepsilon_m \cos m\varphi\right). \tag{5}$$

In an advanced stellarator particle orbit stay closer to the magnetic surfaces than in classical stellarators [7]. Lotz and Nührenberg [8] have evaluated the diffusion coefficient of an advanced stellarator using Monte-Carlo-technique. Their results show a clear reduction of the diffusion coefficient in the Pfirsch-Schlüter regime and in the plateau regime. In these calculations the exact magnetic field represented in Dommaschk potentials [9] is used. As an example of an advanced stellarator we consider a configuration ASC 742 proposed by A. Schlüter. As can be seen from Fig. 1 a typical property of this configuration is the M+S-like deformation and

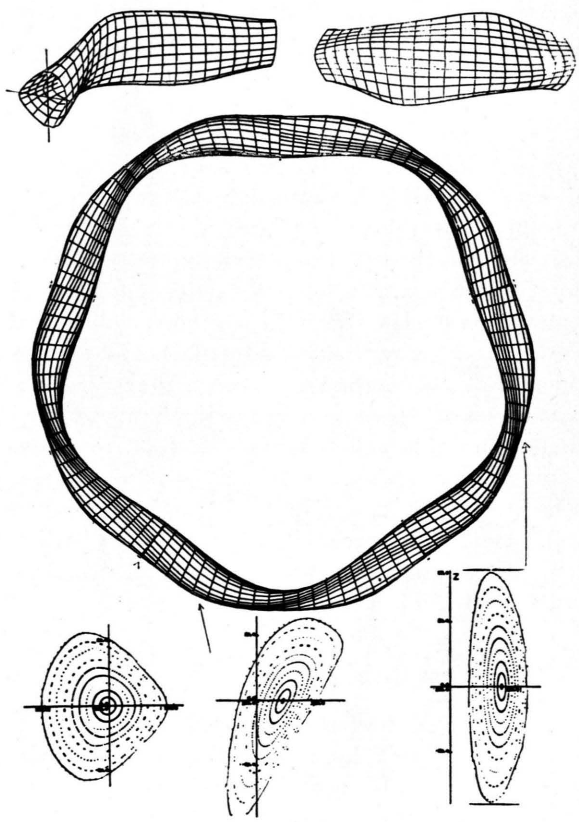

Fig. 1. Top view on a magnetic surface of the configuration ASC 742. 5 periods, rotational transform $\iota = 0.52$. The cross section of the magnetic surfaces at 3 toroidal angles is shown.

the nearly straight sections. Detailed particle orbit analysis has provided the following results: passing particles stay closer to the magnetic surfaces by a factor of 2 compared to classical stellarators, localized particles which are trapped in the center of the straight section drift more slowly in vertical direction as predicted by the usual toroidal drift $v_{\mathrm{Drift}} \approx \varrho/R) v_{\mathrm{th}}$. The drift velocity even vanishes for strongly localized particles. The magnetic field strength on a magnetic surface of ASC 742 can roughly be approximated by (5) with $m=5$. The trapped particles are localized around $m\varphi = k \cdot 2\pi$, $k=0, 1, 2, 3, 4$. In this plane the effective toroidal curvature is proportional to $1-\varepsilon_b$, so that the vertical velocity of strongly localized particles is proportional to $1-\varepsilon_b$. In order to model the advanced stellarator ASC 742 $\varepsilon_b = 1$ and $\varepsilon_m = 0.09$ have been chosen. In case $\varepsilon_b = 0$, $\varepsilon_m \neq 0$ Eq. (5) is a model field for a tokamak with ripple.

The collisions are described by a Lorentz collision operator with the energy of the particles kept constant. All particles are followed until they arrive at the wall, after this they start again on a certain magnetic surface ψ_0. In most cases the position and the pitch angle distribution on the initial magnetic surface is chosen random but the method also allows to model given refuelling mechanisms. The calculation comes to a stationary phase when all particles run through the recycling process at least once, which means after one particle confinement time. From the loss flux in the stationary phase and the number of particles the confinement time is calculated. The method described here requires one particle confinement time as the minimum calculation time whereas in Boozer's method of determining the diffusion coefficient the 90°-scattering time is the minimum. In addition the procedure yields the radial particle distribution, the distribution of escaping particles and the $v_{\|}$-distribution of the particles in the plasma.

III. Numerical Results

The numerical results were obtained with a sample of 500 particles, in some cases also 1000 particles were used. The particles start in the plasma center in most cases, their distribution of the pitch $\Lambda = v_{\|}/v$ is chosen symmetric to $\Lambda = 0$. In the cases presented here the pitch of the starting particles is ± 0.5, the particles are created as passing particles.

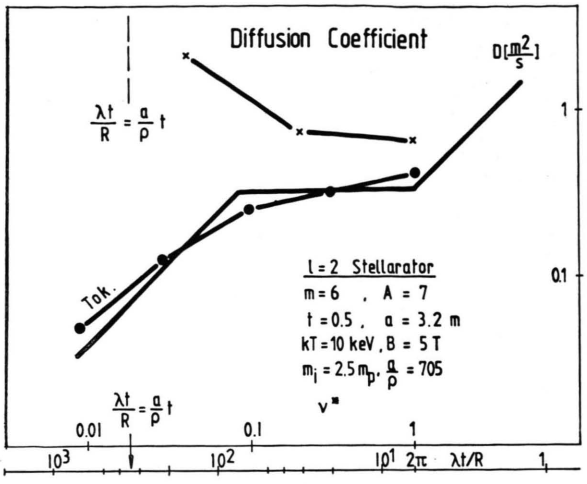

Fig. 2. Diffusion coefficient of monoenergetic particles vs collisionality. $l=2$ stellarator, $m=6$, $A=7$, $\iota=0.5$, $a/\varrho = 705$, $B=5$T.

For comparison also a random distribution function for the starting particles has been chosen, the results only show a small difference.

The diffusion coefficient D has been evaluated in some cases, the results are compared with the loss rate and the confinement time.

Mynick [4] has calculated the ion diffusion coefficient for a stellarator with $m=6$, aspect ratio $A=7$ and reactor dimensions ($B=5\,T$, $a=3.2$ m). In Fig. 2 the diffusion coefficient D is plotted vs collisionality ν^*. The normalized mean free path $\lambda\iota/R$ increases to the left. For comparison also a tokamak case ($\delta_a = 0$) is exhibited. As can be seen from this figure and the figures in Mynick's paper the calculations there are not extended to the long

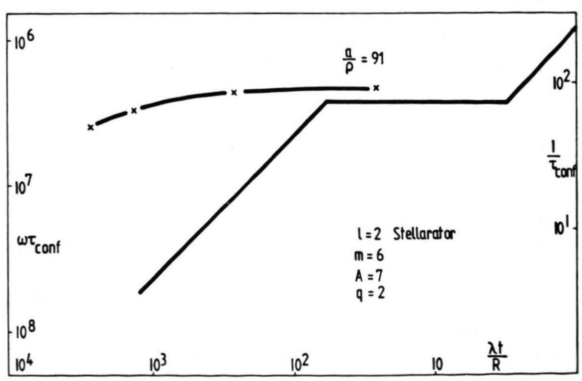

Fig. 3. Loss rate vs $\lambda\iota/R$. $l=2$ stellarator, $m=6$, $\iota=0.5$, $a/\varrho=91$, $1/\tau_{\mathrm{conf}}$ in s^{-1}. Particles start in the center.

mean free path regime where velocity space diffusion begins. The onset occurs at $\lambda\iota/R \approx 350$.

For comparison the same case was calculated for a smaller value of a/ϱ. In Fig. 3 the loss rate $1/\tau_{\mathrm{conf}}$ is calculated up to $\lambda\iota/R > 2000$ under stationary conditions. The parameters of the plasma are: $a = 0.2$ m, $B = 2.1$ T, $\iota = 0.5$, $E = 1.1$ keV, $a/\varrho = 91$. Therefore the onset of velocity space diffusions already happens at $\lambda\iota/R \approx 45$. Figure 3 demonstrates that under these conditions there is no regime of an $1/\nu$-scaling. The analysis of the lost particles shows that the localized particle dominate the loss in the long mean free path regime, particles are lost preferentially in vertical direction which is the direction of the toroidal drift. The $v_\parallel$-distribution of the escaping particles exhibits a peak around $v_\parallel = 0$. Consequently there is a hole in the $v_\parallel$-distribution of the particles in the plasma (Figure 4).

The $v_\parallel$-distribution of plasma particles exhibits a strong asymmetry and more particles with $v_\parallel/v > 0$ than with $v_\parallel/v < 0$ although equal numbers of particles are created at $v_\parallel/v = \pm 0.5$. This asymmetry depends on the collision frequency and vanishes at large collisionality. The asymmetry is caused by the vertical drift of the particles, a reversal of this drift by changing the charge of the particles reversed the sign of the asymmetry. This asymmetry is not a specific property of the stellarator, it also occurs in tokamak configurations (Figure 5).

This asymmetry in the $v_\parallel$-distribution is predicted by neoclassical theory of tokamaks [10], it gives rise to the bootstrap current. Therefore it has to be concluded from the numerical results that in stellarator geometry the same effect happens.

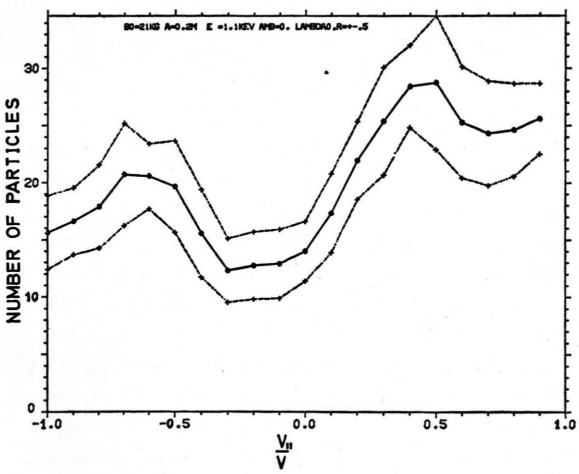

Fig. 4. Distribution of particle vs $v_\parallel/v$. $l = 2$ stellarator, $m = 6$, $A = 7$, $\iota = 0.5$, $\lambda\iota/R = 1.34 \cdot 10^3$, energy $E = 1.1$ keV.

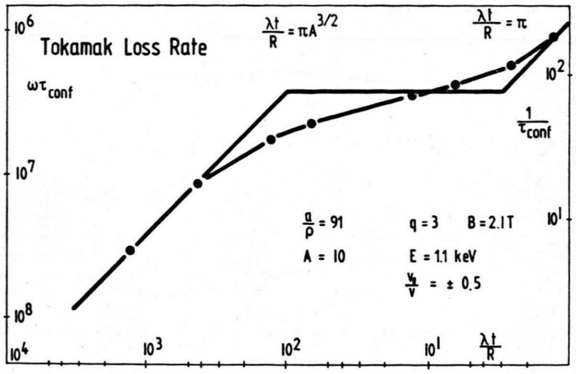

Fig. 5. Loss rate of a tokamak configuration. Energy $E = 1.1$ keV, $A = 10$, $a/\varrho = 91$, $R = 2$ m. $B = 2.1$ T.

If a ripple $\varepsilon_{\mathrm{m}} = 5 - 9\%$ is introduced in a tokamak geometry these localized particles dominate the diffusion losses (Figure 6). In the long mean free path regime $\lambda\iota/R \geqq 10^3$ the value of the ripple ε_{m} is of minor importance, the confinement time is roughly the scattering time into the loss region. Similar as in the stellarator case the loss rate remains in the order of the plateau loss rate and decreases at low collisionality. With increasing value of a/ϱ there is a tendency to an $1/\nu$ scaling (Fig. 7) but even at $a/\varrho = 300$, which is close to reactor conditions, the losses in the long mean free path regime do not increase by more than a factor of two over the plateau value.

At $\lambda\iota/R \geqq 10^4$ the loss rate decreases with λ.

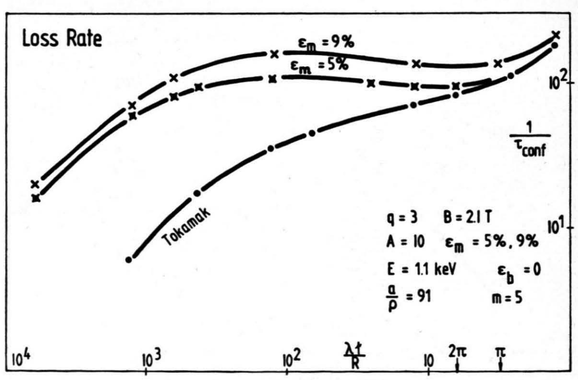

Fig. 6. Loss rate of tokamak configuration with ripple. $A = 10$, $q = 3$, $a/\varrho = 91$, $E = 1.1$ keV, $R = 2$ m.

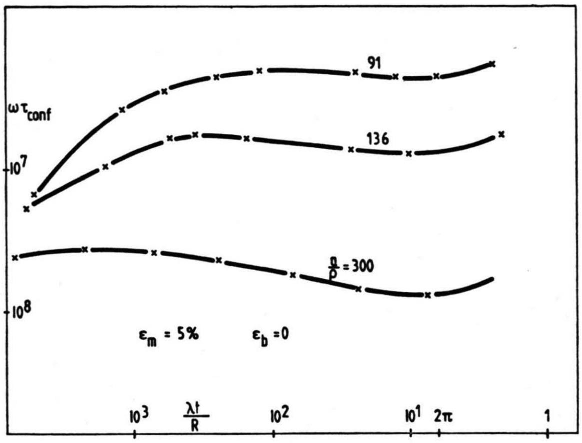

Fig. 7. Configuration same as in Figure 6. $\omega\tau_{\text{conf}}$ vs $\lambda\iota/R$ for different values of a/ϱ. $\varepsilon_{\text{m}} = 0.05$.

In order to model an advanced stellarator $\varepsilon_b = 1$ has been chosen. The parameters of the configuration $A = 10$, $E = 1.1$ keV, $a/\varrho = 91$, are adjusted to the envisaged parameters of Wendelstein VII-AS, an advanced stellarator being planned in Garching.

As shown in Fig. 8 the reduction of the drift of localized particles leads to an appreciable smaller loss rate, the loss rate is even below the plateau value of a tokamak. In case of very frequent collisions in the Pfirsch-Schlüter regime $\lambda\iota/R < \pi$, there is no effect by localized particles. A similar situation occurs at $\lambda\iota/R \geqq 10^4$, in this case the drift time is much smaller than the 90°-scattering time and the loss rate is determined by pitch angle scattering.

The tokamak curve which is used as a reference in Fig. 8 is taken from the corresponding calculation of Figure 5. From dimensional arguments it follows that the confinement time $\omega\tau_{\text{conf}}$ is a function of $\lambda\iota/R$ and a/ϱ (ω = gyrofrequency). In the plateau regime of a tokamak the confinement time has the following scaling

$$\omega\tau_{\text{conf}} \sim \frac{R}{a}\,\iota\left(\frac{a}{\varrho}\right)^3 .$$

The proportionality factor is taken from the numerical results of Fig. 5

$$\omega\tau_{\text{conf}} = \frac{\pi}{2.9}\,\iota\left(\frac{R}{a}\right)\left(\frac{a}{\varrho}\right)^3 . \tag{6}$$

This plateau confinement time can be considered as a lower limit of the confinement time in the advanced stellarator, both in the plateau regime and in the long mean free path regime. If a/ϱ is larger than 400 it has to be expected that in the regime of $\lambda\iota/R = 10^3 - 10^4$ the confinement time is smaller than given by the plateau value.

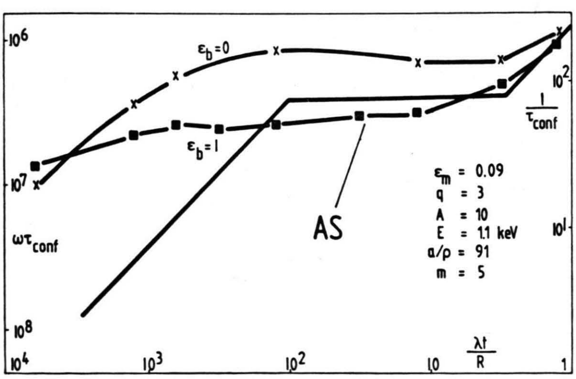

Fig. 8. Loss rate in an advanced stellarator (curve labelled AS). Energy $E = 1.1$ keV, $a/\varrho = 91$, $A = 10$, $q = 3$, $\varepsilon_{\text{m}} = 0.09$ $\varepsilon_{\text{b}} = 1$, $R = 2$ m, $B = 2.1$ T.

IV. Discussion and Conclusions

The present analysis has shown that a stationary plasma with localized particles establishes an anisotropic distribution function in the long mean free path regime. Since the localized particles preferentially leave the plasma a minimum in the distribution function occurs around $v_{\|} = 0$. Consequently the number of localized particles is smaller than in a plasma with an isotropic distribution function. In an isotropic distribution function the fraction of localized particles is of the order $\sqrt{\varepsilon_{\text{h}}}$, the diffusion coefficient is approximately $D \sim \varepsilon_{\text{h}}^{3/2} \cdot v_{\text{D}}^2/\nu$. Diminishing the number of localized particles due to loss cone effects leads to a smaller loss flux. Therefore the $1/\nu$-scaling does not appear in configurations with localized particles if $\lambda/R > a/\varrho$, velocity space diffusion more and more dominates the loss process and in the long mean free path regime the confinement time is of the order of the 90°-deflection time ν^{-1}. The numerical calculations verify the analytical results obtained by Miyamoto [6].

The arguments given above are only relevant for the ions, for electrons a/ϱ is $\sqrt{m_{\text{i}}/m_{\text{e}}}$ times larger. Therefore loss cone behaviour occurs only for high energetic electrons with $\lambda\iota/R > 10^4$ or more. In the regime $\lambda\iota/R = 10^2 - 10^3$ the $1/\nu$-scaling of electron losses has to be expected.

In neoclassical theory the ion losses are faster than electron losses, a radial electric field builds up which makes the diffusion losses ambipolar. The ambipolar electric field reduces the ion losses, thus the loss rates calculated above are upper limits for the particles losses in a real plasma. Also for ion energy losses the loss rate above is an upper limit. The ambipolar electric field mainly reduces the drift of the localized ions and therefore also diminishes the energy loss of ions.

The numerical calculations have assumed constant energy of the particles, but a proper estimate of the ion energy conduction requires also consideration of energy scattering.

Energy scattering has been included in the work of Potok et al. [5]. In this paper the $1/\nu$ scaling of the thermal conductivity was not found. The authors evaluate the conductivity for a configuration with large aspect ratio, the condition $\lambda\iota/R=(a/\varrho)\iota$ leads to $\lambda\iota/R=139$. This value is close to the point where the $1/\nu$-scaling starts. Therefore it has to be concluded that in this configuration the $1/\nu$-scaling of χ_{i} cannot be expected since the basic assumption of an isotropic distribution function is violated. As already shown above this is not the case in the calculations of Mynick [4]. There is no contradiction between the neoclassical theory and the numerical calculations if the conditions for the $1/\nu$-scaling are met. This scaling occurs if

$$\frac{L\iota}{R}\cdot\frac{1}{\varepsilon_{\mathrm{h}}^{3/2}}<\frac{\lambda\iota}{R}<\frac{a}{\varrho}\iota\,. \tag{7}$$

In the configuration investigated by Potok et al. this regime is negligibly small.

The depopulation of the distribution function around $v_{\|}=0$ and the anisotropy might give rise to instabilities and enhanced pitch angle scattering. It has been found that in the case of the advanced stellarator the anisotropy of the particle distribution is smaller than in the conventional stellarator at the same collisionality. This is caused by the reduced drift velocity of the trapped particles. The advantage of the advanced stellarator is twofold: the reduction of the drift velocity leads to smaller radial losses and smaller deviation from isotropy. In addition it is expected that the ambipolar electric field leads to a further reduction of this anisotropy.

Acknowledgements

I am grateful to J. Derr for making available his version of the Boozer-Kuo-Petravic code and for many stimulating discussions. I also should like to thank Mrs. Ott for modifying the code and for her indefatigable assistance during the numerical calculations.

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